

1.2 Arithmetic Series

A series is the sum of the terms of a sequence.

sequence: 1, 2, 3, 4, 5.

Series: $1+2+3+4+5=15$

Quickly add the numbers 1 through 19.

$$1+2+3+4+\dots+17+18+19=190$$

9.5 pairs of 20

$$\underbrace{5+8+11+\dots+44+47+50}_{16 \text{ terms}} = 8 \text{ pairs of } 55 = 440$$

$d=3$
 $t_1=5$

S_n = sum of n terms

ex) Find the sum of $6+11+16+21+\dots+51$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$t_n = t_1 + (n-1)d$$

$$51 = 6 + (n-1)5$$

$$45 = (n-1)5$$

$$9 = n-1$$

$$10 = n$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_n = \frac{10}{2}(6 + 51)$$

$$= 5(57)$$

$$= 285$$

ex) Find S_9 for 10, 12, 14, 16, ... $t_9 = 10 + (9-1)2$

$$S_9 = \frac{9}{2}(10 + 26)$$

$$= \frac{9}{2}(36)$$

$$= 9 \times 18$$

$$= 9 \times 9 \times 2$$

$$= 162$$

(OR)

$$S_n = \frac{n}{2}(t_1 + t_1 + (n-1)d)$$

$$S_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$S_9 = \frac{9}{2}(2(10) + (9-1)(2))$$

$$= \frac{9}{2}(20 + 16)$$

$$= \frac{9}{2}(36) = 162$$

ex) If $S_2 = 19$ and $S_4 = 50$, find t_1, d & S_{20} .

$$S_n = \frac{n}{2}(t_1 + t_n) \quad S_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$19 = \frac{2}{2}(t_1 + t_2)$$

$$\textcircled{1} \quad 19 = t_1 + t_2$$

$$(t_1 + d)$$

$$50 = \frac{4}{2}(t_1 + t_4)$$

$$\textcircled{2} \quad \frac{50}{2} = \frac{2}{2}(t_1 + t_4)$$

$$25 = t_1 + t_4$$

$$t_1 + 3d$$

$$19 = \frac{2}{2}(2t_1 + (2-1)d)$$

$$\textcircled{1} \quad 19 = 2t_1 + d$$

$$50 = \frac{4}{2}(2t_1 + (4-1)d)$$

$$\textcircled{2} \quad \frac{50}{2} = \frac{2}{2}(2t_1 + 3d)$$

$$25 = 2t_1 + 3d$$

$$\textcircled{2} \quad 25 = 2t_1 + 3d$$

$$- \textcircled{1} \quad 19 = 2t_1 + d$$

$$6 = 2d$$

$$\boxed{3 = d}$$

$$19 = 2t_1 + 3$$

$$16 = 2t_1$$

$$\boxed{8 = t_1}$$

$$S_{20} = \frac{20}{2}(2(8) + (20-1)3)$$

$$= 10(16 + (19)3)$$

$$= 10(73) = \underline{730}$$